Cañada College Composition Worksheet

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If you notice any mistakes, please let me know in the STEM Center or email me at ramosj@smccd.edu

1 Composition

Composition means $(f \circ g)(x) = f(g(x))$, where you are inputting the right function into the one to its left. In $f(g(x))$ notation, the outermost function will always be the leftmost function in the $(f \circ g)(x)$ notation.

1. $f(x) = x^2, g(x) = x + 2$, find

   a. $(f \circ g)(x)$

   b. $(g \circ f)(x)$

   c. $(f \circ f)(x)$

   d. $(g \circ g)(x)$

2. $f(x) = x^2 + 1, g(x) = \sin x$, find

   a. $(f \circ g)(x)$

   b. $(g \circ f)(x)$

   c. $(f \circ f)(x)$

   d. $(g \circ g)(x)$

3. $f(x) = x^2 + 1, g(x) = \ln(x + 1)$, find

   a. $(f \circ g)(x)$

   b. $(g \circ f)(x)$

   c. $(f \circ f)(x)$

d. $(g \circ g)(x)$

4. $f(x) = e^x, g(x) = \sin x$, find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

d. $(g \circ g)(x)$

5. $f(x) = \arctan(x^2), g(x) = \cos x$, find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

d. $(g \circ g)(x)$

6. $f(x) = x^2, g(x) = \cos x, h(x) = e^x$, find

a. $(f \circ g \circ h)(x)$

b. $(h \circ g \circ f)(x)$

c. $(g \circ f \circ h)(x)$

d. $(h \circ f \circ g)(x)$

2 Decomposition

Decompose the following functions into 2 or more natural/simple functions:

1. $f(x) = \cos(x^2)$

2. $f(x) = e^{\sin(x)}$

3. $f(x) = [\ln(x)]^3$

4. $g(x) = \arctan(e^x)$

5. $g(x) = \frac{1}{e^x}$

6. $g(x) = \frac{1}{x^2+x+15}$

7. $h(x) = e^{\sin(x^3)}$

8. $h(x) = \ln(\cos(x^5))$

9. $h(x) = e^{\arctan(\ln(x))}$

10. $k(x) = e^{\sin\left(\ln[([\arctan x]^2 + 1)]\right)}$

3   Answers for Composition

1. $f(x) = x^2, g(x) = x + 2$

a. $(f \circ g)(x) = (x + 2)^2$

b. $(g \circ f)(x) = x^2 + 2$

c. $(f \circ f)(x) = x^4$

d. $(g \circ g)(x) = x + 4$

2. $f(x) = x^2 + 1, g(x) = \sin x$

a. $(f \circ g)(x) = (\sin x)^2 + 1$

b. $(g \circ f)(x) = \sin(x^2 + 1)$

c. $(f \circ f)(x) = (x^2 + 1)^2 + 1$

d. $(g \circ g)(x) = \sin(\sin x)$

3. $f(x) = x^2 + 1, g(x) = \ln(x + 1)$

a. $(f \circ g)(x) = (\ln(x + 1))^2 + 1$

b. $(g \circ f)(x) = \ln([x^2 + 1] + 1)$

c. $(f \circ f)(x) = (x^2 + 1)^2 + 1$

d. $(g \circ g)(x) = \ln(\ln(x + 1) + 1)$

4. $f(x) = e^x, g(x) = \sin x$

a. $(f \circ g)(x) = e^{\sin x}$

b. $(g \circ f)(x) = \sin(e^x)$

c. $(f \circ f)(x) = e^{e^x}$

d. $(g \circ g)(x) = \sin(\sin x)$

5. $f(x) = \arctan(x^2), g(x) = \cos x$

a. $(f \circ g)(x) = \arctan[(\cos x)^2]$

b. $(g \circ f)(x) = \cos(\arctan(x^2))$

c. $(f \circ f)(x) = \arctan([\arctan(x^2)]^2)$

d. $(g \circ g)(x) = \cos(\cos x)$

6. $f(x) = x^2, g(x) = \cos x, h(x) = e^x$

a. $(f \circ g \circ h)(x) = [\cos(e^x)]^2$

b. $(h \circ g \circ f)(x) = e^{\cos(x^2)}$

c. $(g \circ f \circ h)(x) = \cos(e^{2x})$

d. $(h \circ f \circ g)(x) = e^{(\cos x)^2}$

4 Answers for Decomposition

Answers are ordered from innermost function to outermost.

1. $f(x) = \cos(x^2) \implies g(x) = x^2, h(x) = \cos(x)$

2. $f(x) = e^{\sin(x)} \implies g(x) = \sin(x), h(x) = e^x$

3. $f(x) = [\ln(x)]^3 \implies g(x) = \ln(x), h(x) = x^3$

4. $g(x) = \arctan(e^x) \implies f(x) = e^x, h(x) = \arctan(x)$

5. $g(x) = \frac{1}{e^x} \implies f(x) = e^x, h(x) = \frac{1}{x}$

6. $g(x) = \frac{1}{x^2+x+15} \implies f(x) = x^2 + x + 15, h(x) = \frac{1}{x}$

7. $h(x) = e^{\sin(x^3)} \implies f(x) = x^3, g(x) = \sin(x), k(x) = e^x$

8. $h(x) = \ln(\cos(x^5)) \implies f(x) = x^5, g(x) = \cos(x), k(x) = \ln(x)$

9. $h(x) = e^{\arctan(\ln(x))} \implies f(x) = \ln(x), g(x) = \arctan(x), k(x) = e^x$

10. $k(x) = e^{\sin\left(\ln[([\arctan x]^2 + 1)]\right)} \implies f(x) = \arctan(x), g(x) = x^2 + 1, h(x) = \ln(x),$
$r(x) = \sin(x), s(x) = e^x$