ALGEBRA

PROPERTIES

ARITHMETIC PROPERTIES

ASSOCIATIVE	$a(bc) = (ab)c$
COMMUTATIVE	$a + b = b + a$ and $ab = ba$
DISTRIBUTIVE	$a(b + c) = ab + ac$

ARITHMETIC OPERATIONS EXAMPLES

$ab + ac = a(b + c)$

$a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

$\frac{a - b}{c - d} = \frac{b - a}{d - c}$

$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{ab + ac}{a} = b + c, a \neq 0$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$

QUADRATIC EQUATION

For the equation

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RADICAL PROPERTIES

$a, b \geq 0$ for even n

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

EXPONENT PROPERTIES

$a^n a^m = a^{n+m}$

$(a^n)^m = a^{nm}$

$(ab)^n = a^n b^n$

$a^{-n} = \frac{1}{a^n}$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

$a^0 = 1, a \neq 0$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\frac{1}{a^{-n}} = a^n$

$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$

PROPERTIES OF INEQUALITIES

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $a/c < b/c$

If $a < b$ and $c < 0$ then $ac > bc$ and $a/c > b/c$

PROPERTIES OF COMPLEX NUMBERS

$i = \sqrt{-1}$

$i^2 = -1$

$\sqrt{-a} = i\sqrt{a}, \quad a \geq 0$

$(a + bi) + (c + di) = a + c + (b + d)i$

$(a + bi) - (c + di) = a - c + (b - d)i$

$(a + bi)(c + di) = ac - bd + (ad + bc)i$

$(a + bi)(a - bi) = a^2 + b^2$

$|a + bi| = \sqrt{a^2 + b^2}$

$\overline{(a + bi)} = a - bi$

$\overline{(a + bi)}(a + bi) = |a + bi|^2$

$\frac{1}{(a + bi)} = \frac{(a - bi)}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}$

LOGARITHM PROPERTIES

if $y = \log_b x$ then $b^y = x$

$\log_b b = 1$ and $\log_b 1 = 0$

$\log_b b^x = x$

$b^{\log_b x} = x$

$\log_a x = \frac{\log_b x}{\log_b a}$

$\log_b(x^r) = r \log_b x$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

COMMON FACTORING EXAMPLES

$x^2 - a^2 = (x + a)(x - a)$

$x^2 + 2ax + a^2 = (x + a)^2$

$x^2 - 2ax + a^2 = (x - a)^2$

$x^2 + (a + b)x + ab = (x + a)(x + b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

ABSOLUTE VALUE

$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$

$|a| = |-a|$

$|a| \geq 0$

$|ab| = |a||b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a + b| \leq |a| + |b|$

The STEMCENTER at Cañada College

COMPLETING THE SQUARE

$ax^2 + bx + c = a(\ldots)^2 + \text{constant}$

1. Divide by the coefficient a.
2. Move the constant to the other side.
3. Take half of the coefficient b/a, square it and add it to both sides.
4. Factor the left side of the equation.
5. Use the square root property.
6. Solve for x.