

# TRIGONOMETRY

# LAWS AND IDENTITIES

## TANGENT IDENTITIES

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

## RECIPROCAL IDENTITIES

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

## PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

## PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta$$

$$\tan(\theta + \pi n) = \tan \theta$$

$$\csc(\theta + 2\pi n) = \csc \theta$$

$$\sec(\theta + 2\pi n) = \sec \theta$$

$$\cot(\theta + \pi n) = \cot \theta$$

## EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$

## DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

## HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

## LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

## PRODUCT TO SUM IDENTITIES

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

## SUM TO PRODUCT IDENTITIES

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

## LAW OF SINES

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

## LAW OF TANGENTS

$$\frac{a - b}{a + b} = \frac{\tan\left[\frac{1}{2}(\alpha - \beta)\right]}{\tan\left[\frac{1}{2}(\alpha + \beta)\right]}$$

$$\frac{b - c}{b + c} = \frac{\tan\left[\frac{1}{2}(\beta - \gamma)\right]}{\tan\left[\frac{1}{2}(\beta + \gamma)\right]}$$

$$\frac{a - c}{a + c} = \frac{\tan\left[\frac{1}{2}(\alpha - \gamma)\right]}{\tan\left[\frac{1}{2}(\alpha + \gamma)\right]}$$

## SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

## MOLLWEIDE'S FORMULA

$$\frac{a + b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha - \beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$

## Power Reduction Identities

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2(\theta) = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

## COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

The  
**STEMCENTER**  
at Cañada College